Differential Equation Ch3.1

Q16.

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自動產生的描述

since e^(-t) approaches to 0 when t->inf, so the only way to make the solution approaches to 0 is to make (2+a)/3 = 0

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Q17一張含有 文字 的圖片

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after solving the characteristic equation, we get the following result:

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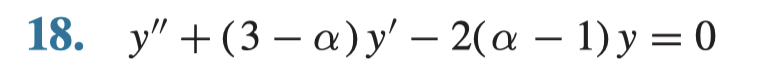
Now, it is obvious that when a<0,t->inf, y->0

However, for a = 1, only c\_1\*e^(at) go unbounded, and [c\_2\*e^(a-1)t] is bounded.

But the question is asking all(nonzero) solution become unbounded, so if c\_1 = 0, we still get a bounded solution

So, the answer for t->inf, y->inf is a>1.

Q18.



after solving, we have:

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we get y->0 as t->inf if a<1

For solution of a where all non-zero solutions become unbounded, since there is c\_2e^(-2t) term, which means that if c\_1 = 0, there is always a bounded solution.

Therefore the solution for above statement is none.

q20

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for part (a), since there is three cases for the characteristic equation(two real roots, repeated roots and imaginary roots), so it cannot find equilibrium solution based on characteristic equation method.

Then, we want to find some constant solution. And we observe that if y = d/c, it does not depends on t, so y’’ = y’ = 0, and cy = c(d/c) = d

so the equilibrium solution/ constant solution is y = d/c

For part (b), Y = y – y\_e , since y\_e = d/c, thus Y = y – d/c, Y’’ = y’’, Y’ = y’

so,

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(don’t know how to come up with this solution…)

Q21.

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it is the general question to sum up this chapter

first write and solve the characteristic equation: ar^2 + br + c = 0, and thus r = 一張含有 文字 的圖片

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Then, use the analysis skills learnt in high school:

a), for root to be real and different: Delta >0

b^2 – 4ac > 0

b^2 > 4ac

for root to be negative: since sqrt(x) > 0

so the only way to make root to be both negative is: -b <0 and -b + sqrt(b^2 – 4ac) < 0

-b < - sqrt(b^2 – 4ac) and b > 0

b > sqrt(b^2 – 4ac) and b>0

b^2 > b^2 – 4ac and b>0

4ac > 0

so, 0 < 4ac < b^2 and b>0

so, 0 < c < b^2/4a and b>0 (since a>0)

b). for root to be real and opposite sign:

Real and different(opposite sign already means different!!) : b^2 – 4ac > 0

b^2 > 4ac

Opposite sign:

condition 1: - b + sqrt(b^2- 4ac) > 0 , condition 2: -b – sqrt(b^2-4ac) < 0

=> sqrt(b^2-4ac) > b => - sqrt(b^2-4ac) < b

=> b^2 – 4ac > b^2 => b^2-4ac < b^2

so -4ac > 0

since a > 0, c < 0(the only requirement)

c).

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自動產生的描述